HEDGE FUND ANALYSIS – A GLANCE INTO THE »BLACK BOX«

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1. INTRODUCTION

The target of this analysis – based on the example of one hedge fund strategy – is to show which market factors (e.g. equity and bond indices and their volatilities) explain the return of hedge funds and which modeling types are necessary to guarantee a high quality explanation. Empirical methods, e.g. classical linear regression; extended models also include autocorrelation (dependency on residuals) and heteroscedasticity (non-constant volatility of residuals over a period of time), are used to investigate which variables (factors) are relevant for explaining hedge fund returns and to determine the quality of the model. The results show that strategies which can be significantly more complex within the hedge fund business than in traditional asset management can be described with only a few significant market factors.

2. DATA

The data used consists of the time series of return of CSFB/Tremont Fixed Income Arbitrage Index to be analyzed (regressand) and 26 time series of market data (regressors). All time series consist of monthly data from January 1994 to November 2002. Market data contains time series of return of equity and bond indices, volatility of equity and bond indices as well as time series of differences in return between corporate bond indices (rating AAA to C) and US government bonds. The difference in return is the monthly increase or decrease in return of corporate bonds compared to government bonds, which results from higher interest coupons and changes of rating (changes of risk premium). Costs and returns from leverage or arbitrage strategies as well as time series of return of option strategies could be modeled as additional reasonable factors, which can certainly lead to a better explanation. Such factors are not included in the following calculations.

3. CLASSICAL LINEAR REGRESSION METHOD

The first model applied is the classical linear regression where a regressand $y$ (here return of the Hedge Fund strategy) is explained through one or more regressors $x_i$. A linear coherence between regressand and regressors is assumed.

$$y = \alpha + \beta_1 \cdot x_1 + \beta_2 \cdot x_2 + \ldots + \beta_n \cdot x_n + \epsilon$$  

or

$$y = \beta X + \epsilon$$

The following questions should be answered using this approach:

- How important is the influence of the considered variables (regressors)?
Do these variables have a significant influence at all, e.g. do they really contribute to an explanation of the variance of the regressands?

Which variables out of 26 (factors) should be considered (selection of variables)?

The searched value for the parameter $\alpha$ and $\beta$ are estimated – given the usual assumptions of the classical linear model which will be discussed later – using the Least Squares Method.

In order to guarantee the exclusive use of significant factors for the estimation model we have to define a mechanism which helps to identify the best model. Theoretically there are $2^{N+1}$ possible regression models given N+1 regressors (including the constant $\alpha$). Fahrmeir and Hamerle (1984) describe methods on how to execute a complete search for the best model. We restrict ourselves here to a step-by-step selection process. Principally speaking, there are two methods: forward selection (starting with a model that only contains $\alpha$ and additionally choosing the variable with the highest level of significance using a step-by-step method) and backward elimination (starting from the complete model and eliminating variables until only those variables with a predetermined level of significance remain in the model). We are applying the second mechanism. From the initial 26 regressors, we eliminate variables until only significant regressors remain in the model. The following results contain a level of significance of at least 95%.

After choosing the optimal model, the variable $y$ can be divided in three components. The absolute term of the regression is $\alpha$ which has a constant size at any time independent of the regressors; without the influence of additional regressors the regressand would always have the value $\alpha$. The size of the influence of N regressors is expressed via the value of $\beta$. An important goal of regression analysis is to reveal the systematic influence of factors on the explained variable. In addition to the systematic variation of the regressand, which can be explained through the trends of other variables, there is the third component, called unexplainable variations, which can be displayed through the error variable (also called the interfering variable) $\varepsilon$.

**RESULTS OF THE REGRESSION**

The hedge fund strategy Fixed Income Arbitrage is analyzed with the above mentioned method, as well as the chosen selection mechanism. The return of this strategy showed an annual average of 6.5% p.a. (0.53% per month).
The estimated parameters of the regression analysis are summarized below.

<table>
<thead>
<tr>
<th>Factor (regression variable)</th>
<th>Value of Alpha or Beta</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.843 (% per month)</td>
<td></td>
</tr>
<tr>
<td>Monthly difference in the return of US-Corporate Bonds (Rating A) and US-Government Bonds (Indices of Bonds with 7-10 years duration)</td>
<td>+0.581</td>
<td>The positive Beta proves that the strategy is «credit long» and excess returns can be generated on declining spreads</td>
</tr>
<tr>
<td>Return of the US-Corporate Bond-Index (Bonds with 7-10 years duration)</td>
<td>+0.157</td>
<td>The strategy has a long position on interest rates</td>
</tr>
<tr>
<td>Absolute monthly change of volatility of US-Standard-Equity-Index</td>
<td>-0.103</td>
<td>Volatility variation leads to losses of the hedge fund strategy.</td>
</tr>
</tbody>
</table>

From a theoretical point of view (see Inneichen, 2000) the value and algebraic sign of Beta for the variables – with the exception of the equity-volatility-factor – seem to be plausible. For the latter it seems generally implausible that this factor should be relevant for a Fixed Income Arbitrage strategy.

The coefficient of determination $R^2$ of 0.205 signifies that the explanatory quality of this model is rather low, which may result from missing factors that could improve the model.

**PROBLEMS OF THE REGRESSION ANALYSIS**

Generally speaking, it is obvious that the explanatory quality of the above mentioned model is rather low. Possibly, missing factors are the reason for this. Therefore, in the next section, extended models will be applied.

Additional significant problems result from the assumptions of the disturbance $\varepsilon$, which need to be met when the classical linear regression is applied. Concerning the error term $\varepsilon$, the following distribution assumptions have to be fulfilled:

Nonautocorrelation: $\text{Cov}(\varepsilon_i, \varepsilon_j) = 0$

Homoscedasticity: $\text{Var}(\varepsilon_i) = \sigma^2$

This is – as shown below – not the case. In fact the residuals within the executed regression are correlated and the volatility of the disturbance is not constant over time. Also, the regressand must not show a time trend – technically it has to be stationary; otherwise the regression
shows an excessive significance level. The below graphics show how the
time series of the residuals (upper graphics) respectively of the
regressand (lower graphics) develop under a violation of these
assumptions.

**Autocorrelated residuals**

![Autocorrelated residuals graph]

This graph makes it clear that the residual at a point \( t \) influences the
disturbance at the point \( t+1 \),
which violates the assumption of non-autocorrelation.

**Heteroscedastic residuals**

![Heteroscedastic residuals graph]

This graph shows diverging
errors in time, which proves that
the variance of residuals is not constant over time and therefore
the assumption of homoscedasticity is violated.

**Stationary time series**

![Stationary time series graph]

Series without time trend, e.g. returns

**Non-stationary time series**

![Non-stationary time series graph]

Series with time trend, e.g. Indices

**MONITORING OF THE ASSUMPTIONS FOR THE FIXED INCOME ARBITRAGE INDEX**

The below graphic shows the standardized error terms vs. the time series of the CSFB/Tremont Fixed Income Arbitrage Index. The chart clarifies a connection between the disturbance and the time series, which provides an indication of a possible autocorrelation of the residuals. The scattergram does not give a specific indication about heteroscedasticity.
The Durbin-Watson-Test for monitoring of the autocorrelation of a time series can show – at a test statistic of 1.401 – with high significance that the time series of CSFB/Tremont Fixed Income Arbitrage Index is auto-correlated. In addition, the Augmented-Dickey-Fuller-Test can prove that the time series is highly stationary, which can be expected from any return time series. Generally speaking, the classical linear regression is not suitable to identify those risk factors that influence the return of hedge funds. The problem is that the underlying time series as basis and the chosen method lead to the violation of assumptions which have to be fulfilled in order to generate a useful interpretation of the results.

4. EXTENDED MODELS

BASICS

In order to get a grip on autocorrelation and heteroscedasticity, the linear basis model has to be extended by two components. Autoregression is modeled as an AR(1)-Process, e.g. the residuals in a period are not independent, instead they are dependent on the previous period.

The autoregressive model (AR(1)) is:

\[ y_t = \beta X_t + \varepsilon_t \]
\[ \mu_t = \rho \mu_{t-1} + \varepsilon_t \]

The heteroscedasticity – i.e. the characteristic that volatility of the disturbance is not constant over time – is captured with an ARCH(1)-Model (Autoregressive Conditional Heteroscedasticity).
\[
    y_t = \beta X_t + \epsilon_t \\
    \sigma^2_t = \gamma_0 + \gamma_1 \epsilon_{t-1}^2 \\
    \epsilon \sim N(0, \sigma^2_t)
\]

We use the same selection mechanism with this extended model as with the previous approach.

**RESULTS OF THE REGRESSION**

With the extended model adopted and the chosen selection mechanism, the Fixed Income Arbitrage hedge fund strategy is analyzed. The average return of this strategy is 6.5% p.a. (0.53 % per month).

The estimated parameters of the regression analysis are summarized below:

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<th>Factor (regression variable)</th>
<th>Value of Alpha or Beta</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1.12 (% per month)</td>
<td></td>
</tr>
<tr>
<td>Volatility of US-corporate bond-Index (bonds with a 5 year duration)</td>
<td>-0.146</td>
<td>Vega short, higher market uncertainty leads to losses with this strategy</td>
</tr>
<tr>
<td>Monthly return difference between US-corporate bonds (Rating AA) and US-government bonds (Indices of bonds with 7-10 years duration)</td>
<td>-0.584</td>
<td>Positive betas express that this strategy is long in the respective rating class; negative betas therefore mean the opposite. This strategy also takes credit risks in lower rating classes (A and B) and finances these positions through short positions in less risky rating class AA which also hedges the interest rate risk. Excess returns are generated in case the spreads between the rating classes get smaller.</td>
</tr>
<tr>
<td>Monthly return difference between US-corporate bonds (Rating A) and US-government bonds (Indices of bonds with a 7-10 years duration)</td>
<td>+0.636</td>
<td></td>
</tr>
<tr>
<td>Monthly return difference between US-corporate bonds (Rating B) and US-government bonds (Indices of bonds with a 7-10 years duration)</td>
<td>+0.108</td>
<td></td>
</tr>
<tr>
<td>Monthly return difference between US-corporate bonds (Rating C) and US-government bonds (Indices of bonds with a 7-10 years duration)</td>
<td>-0.036</td>
<td></td>
</tr>
</tbody>
</table>
From a theoretical point of view (see Inneichen 2000) the value and algebraic sign of the betas for the variables seem plausible. In particular, the positions within the rating classes (with the exception of the very low beta for the rating class C) correspond highly with the theoretic common assumptions in the Fixed Income Arbitrage strategy.

Furthermore, the results of this regression show that the modeling of the autocorrelation (AR(1)-Process) as well as of the heteroscedasticity (ARCH(1)-Model) is adequate.

5. SUMMARY
MISSING FACTORS
For both regressions we can see that the calculated alpha is higher than the monthly average return. This means that the contribution of the risk factor to the overall return of the strategy is negative. The factors are therefore explicit cost factors and not return factors. However, this does not appear implausible. As already mentioned, no arbitrage factors or option strategies have been modeled. For both it can be assumed that, by using this strategy, regular returns can be generated at the cost of losses that are higher than the monthly premium during periods of high stock exchange volatility. Through the regressors, the return-generating factors are not displayed – as opposed to the loss-generating factors, which are displayed (e.g. volatility).

OPEN TOPICS
The monitoring of the stability of these results is the first task that suggests itself when examining hedge fund returns. Stability over time should be examined, e.g. via the partition of the overall period into two segments. It should also be analyzed whether the method is applicable for indices of other providers or for single hedge funds, and whether the results are comparable.

APPLICATION OF RESULTS
The insights into which factors are significant and what influence they have on the return of hedge fund strategies can be adopted within risk analysis. Based on the calculated market exposure and the known/forecasted market movement, the portfolio can be simulated on a generic basis and standard risk analysis (e.g. VaR) can be performed. Also, correlations between market factors can be considered on the portfolio level. Using information on relevant market factors, even more specific diversification is possible.

6. LITERATURE