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In Search of the Optimal Fund of Hedge Funds

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Abstract

In this paper we investigate whether it is possible for a fund of hedge funds to not only offer investors access to a diversified basket of hedge funds but to provide skewness protection at the same time. We study two different strategies. The first is for a fund to buy stock index puts and leverage itself, in line with the skewness reduction strategy proposed earlier in Kat (2002). In general, the latter strategy is too dependent on the actual asset allocation strategy followed by investors to allow a fund to be constructed that is optimal for all investors at the same time. However, for investors that invest more or less equal amounts in stocks and bonds and who keep their hedge fund allocation below 30% such a fund can indeed be structured. The second strategy is for a fund to buy put options on itself. We show that this does allow a fund to offer skewness protection to different types of investors at the same time, but compared to the optimal strategy the protection will be somewhat less accurate. Under both strategies the fund of funds is likely to incur a significant loss in expected return. As long as the hedge fund allocation stays below 30%, however, the loss of expected return on investors' overall portfolios will remain limited.

1. INTRODUCTION

Hedge funds can significantly improve a portfolio's mean-variance characteristics but tend to do so at the cost of significantly lower skewness. For investors that use hedge funds in risk reduction or yield enhancement strategies it is therefore important to know how to hedge the drop in skewness that can be expected when hedge funds are added to a portfolio. In Kat (2002) we discussed one possible hedging strategy and showed that by buying out-of-the-money index puts combined with leverage investors can eliminate the unwanted skewness effects of hedge funds at a reasonable price. Although quite straightforward, for many investors the strategy advocated will present additional complications they would rather do without. With most investors investing in funds of hedge funds these days, the next question therefore is whether it is possible to structure a fund of hedge funds that not only offers investors an easy way into hedge funds but which at the same time provides the required skewness protection.

Since much depends on the interaction between stocks and hedge funds, the skewness reduction strategy of Kat (2002) depends heavily on how the investor allocates between stocks and bonds and the relative size of the hedge fund allocation. Since different investors will allocate differently, the optimal skewness reduction strategy will therefore typically be different as well. However, there are special cases in which the optimal skewness reduction strategy is similar for investors following different allocation strategies. In that case it is indeed possible to create one single fund of hedge funds that is optimal for different types of investors at the same time. In the first part of this paper we discuss one of these cases in more detail. A second strategy that would allow a fund of hedge funds to offer skewness protection is for the fund to simply buy put options on itself. Doing so will sever the fund's link with the stock market in case the former is dragged down by a drop in the latter. We discuss this strategy in the second part of the paper.

2. The Data

For our data we return to Kat (2002), meaning that we again distinguish four different asset classes: stocks, bonds, hedge funds and out-of-the-money puts. Stocks are represented by the S&P 500 index, bonds by the 10-year Salomon Brothers Government Bond index and hedge funds by the median equally-weighted portfolio of 20 different individual funds. The returns on out-of-the-money puts are taken to be the returns on a monthly rollover strategy where on the first trading day of the month we buy a slightly out-of-the-money S&P 500 put that expires the next month, on the first trading day of the next month we sell the latter option and replace it with another S&P 500 put that expires a month later, etc. More details on the basic return characteristics of these four asset classes can be found in Kat (2002, table 1).

3. 50/50 Investors

Let's assume we are dealing with investors who always invest and equal amount in stocks and bonds. We refer to such investors as '50/50 investors'. When adding hedge funds to their portfolio, 50/50 investors will reduce their stock and bond holdings by the same amount. This gives rise to portfolios like 45% stocks, 45% bonds and 10% hedge funds or 40% stocks, 40% bonds and 20% hedge funds. One reason to follow a strategy like this might be that these investors are aiming to reduce the risk of their portfolio without wanting to give up expected return. If we were to derive our expectations from return data over the period 1994-2001, the average hedge fund has an expected return more of less equal to that of an equally weighted portfolio of stocks and bonds. Replacing stocks and bonds by hedge funds in a 50/50 way will therefore have little impact on the portfolio's expected return. Since hedge funds are only weakly correlated with stocks and bonds, however, the overall risk (as measured by the standard deviation) will drop.

<< Insert Table 1 >>

Starting with a portfolio of 50% stocks and 50% hedge funds we introduced hedge funds in 50/50 investors' portfolios in 5% steps. The resulting portfolios and their basic return statistics, i.e. mean, standard deviation, skewness and kurtosis, can be found in table 1. From table 1 we see that introducing hedge funds indeed lowers the standard deviation of the portfolio return distribution, but it also reduces the distribution's skewness. What happens to the mean depends very much on the assumption we make for the stock return. If we optimistically assume that the expected return on stocks equals the average return over the sample period June 1994 – May 2001 (1.46%) then the mean stays more or less unchanged. However, if we assume a significantly lower expected return for stocks, things are different. With an expected return on stocks of only 1% the addition of hedge funds will raise the portfolio's expected return quite substantially. Of course, this assumes that the expected return on hedge funds does equal the average return over the sample period. With stock markets coming down, and the number of hedge funds and the total amount of assets under management substantially increasing, one might argue that this will not be the case either. However, at the moment we know just too little about hedge funds to be able to make a realistic alternative assumption. We will therefore continue under the assumption made, which should not provide too much of a problem as in this paper we primarily concentrate on risk and not expected return.

<< Insert Table 2 and 3 >>

4. SKEWNESS REDUCTION

In Kat (2002) we showed that to eliminate the additional negative skewness resulting from the inclusion of hedge funds in a portfolio, all that investors need to do is to allocate a small fraction of wealth to out-of-the-money stock index put options. Doing so will not only restore the skewness of the portfolio return distribution but it will also reduce its standard deviation. If this is not desired, it is easily corrected by applying some leverage to the portfolio. For different initial hedge fund allocations, table 2 shows the allocations resulting after the above skewness reduction strategy has been implemented. The return statistics of these portfolios can be found in table 3. From the latter table we clearly see that after the implementation of the skewness reduction strategy, the skewness of these portfolios' return distributions is back to the same level as without hedge funds (-0.33) and the distributions' kurtosis is much closer to zero. The standard deviations, however, are equal to those of portfolios with hedge funds. In other words, *by buying puts and leveraging the resulting portfolio investors can eliminate the additional skewness and kurtosis but preserve the reduction in the standard deviation that results from the inclusion of hedge funds*. As discussed in more detail in Kat (2002), this of course comes at a cost in the form of a loss of some expected return. Under two different assumptions with respect to the expected return on stocks and assuming investors can leverage at a 4% interest rate, the third and fifth column of table 3 show that the reduction in overall expected return (expressed on a per annum basis) due to the skewness reduction strategy will be relatively limited. This is good news, especially for the many private investors to whom the additional skewness introduced by hedge funds presents a serious danger of not being able to realize their long term goals.

5. FUNDS OF HEDGE FUNDS WITH SKEWNESS PROTECTION

We are now in a position to structure a range of funds of hedge funds that not only invest in hedge funds but which at the same time provide different types of investors (as defined by their desired hedge fund allocation) with the required skewness reduction strategy. Suppose we created a very special type of fund of funds that required no investment from the investor at all. The investor would simply subscribe to the fund for a notional amount equal to the amount he wanted to invest in hedge funds and at the end of the month the fund would pay him a return on that notional depending on the net performance of the fund's assets and liabilities.

<< Insert Table 4 >>

With no money coming in, the fund would of course have to finance itself by borrowing. To determine how much the fund should borrow and how it should invest the proceeds we can follow a similar line of reasoning as advocated in Kat (2001). First, we look what portfolio the investor really wants. This information can be found in table 2. Second, we look at what portfolio the investor will actually be holding. Since all hedge fund exposure will come from the fund and the fund requires no investment from the investor, the investor can simply maintain his original portfolio of 50% stocks and 50% bonds. Third, we calculate the difference between the portfolio that the investor wants and the portfolio that he will actually hold. For the various hedge fund allocations, the resulting fund of funds portfolios (with allocations expressed as a percentage of the amount subscribed in the fund) are shown in table 4 under 'Zero Invest'. From the table we see that the funds will borrow money, short stocks and bonds and invest the proceeds in hedge funds and puts.

Although the above funds of funds range will do the trick, investors may have problems coming to grips with the admittedly somewhat unusual 'zero investment' concept. In addition, since we are essentially talking about a swap contract here, it may well be that many investors are simply not allowed to invest in it. This means we have to structure something a little more conventional instead. Instead of assuming that investors simply hang on to their original 50/50 portfolio of stocks and bonds we could, more traditionally, require them to liquidate part of their portfolio and invest that money with the fund. Part of their money would be in stocks and bonds and the remainder would be invested in the fund. Let's assume investors invested as much in the fund as they originally wanted to invest in hedge funds. A 50/50 investor who wanted to invest 20% in hedge funds for example would liquidate 20% of his stock holdings and 20% of his bond holdings and pass the proceeds on to the fund. As a result, 80% of his total asset value would be in stocks and bonds (still in equal proportions) and 20% would be in the fund. Under this assumption the required fund allocations for the different types of investors (expressed as a percentage of the amount invested in the fund) are shown in table 4 under 'Full Invest'. From the table we see that this time the fund is not short but long stocks and bonds. The allocations to hedge funds, puts and cash are the same as before because, apart from the fact that the fund does not need to neutralize part of the investor's stock and bond holdings now, in essence nothing has changed.

<< Insert Table 5 >>

Although in general the required allocations depend on the type of investor a fund is aimed at, from table 4 we see that for investors choosing hedge fund allocations ranging from 5-20% the optimal fund of funds portfolios are remarkably similar. For all these investors the optimal 'zero investment' fund consists of around –14.5% in stocks, -14.5% in bonds, 111.35% in hedge funds, 2.65% in puts and –85% in cash, while the optimal 'full investment' fund would consist of around 35.5% in stocks and 35.5% in bonds with the same hedge fund, put and cash allocations. To confirm this, we calculated the usual return statistics for different combinations of stocks, bonds and hedge funds, assuming investors invest in the full investment fund (note, however, that by construction investing in the zero investment fund would produce exactly the same results). The results can be found in table 5 under 'Realized'. Comparing the entries under 'Realized' with those under 'Target', which are simply taken from table 3, we see that for relatively low hedge fund allocations our 'optimal' fund of funds performs very well. Irrespective of the hedge fund allocation, the realized loss in expected return, standard deviation, skewness and kurtosis are very similar to the target values. For hedge fund allocations exceeding 30% the difference increases, however. For the latter, the standard deviation and skewness turn out higher and kurtosis lower than desired.

Two points are interesting to note. First, although the effect on the expected return of the overall portfolio appears relatively small, purchasing stock index puts and leveraging the resulting portfolio can under some conditions have a strong impact on the expected fund return. With an expected stock return of 1.46%, the expected fund return drops from 11.9% per annum without puts and leverage to a similar 11.3% with. However, with an expected return on stocks of 1%, the expected fund return drops much further to 9.4%. Second, the optimal fund of funds is required to borrow quite some money. This means that the fund's funding rate is an important variable. This in turn provides fund operators that are able to fund at a better rate with a solid opportunity to add some real value (or take some extra margin).

<< Insert Table 6 >>

6. THE CASE OF 33/66 INVESTORS

The above confirms that for 50/50 investors who allocate less than one third of their asset value to hedge funds it is indeed possible to design a single fund of hedge funds that provides optimal build-in skewness protection. Unfortunately, this conclusion does not automatically extend to other types of investors. Consider, for example, the case of 33/66 investors. These are investors that divide the money invested in stocks and bonds in such a way that 1/3 is invested in stocks and 2/3 is invested in bonds. Now suppose we repeated the above exercise assuming we were dealing with 33/66 investors instead of 50/50 investors. On one hand the results would be similar, but on the other they would not. Under the assumptions made, the return distribution of a portfolio of 1/3 stocks and 2/3 bonds will have a skewness of 0.03, i.e. be more or less symmetrical, which could be an important reason to hold such a portfolio in the first place. When introducing hedge funds, this changes quickly though. With 10% hedge funds in the portfolio skewness drops to –0.14, with 20% to -0.34, with 30% to –0.52 and with 40% to –0.66. This means that for 33/66 investors skewness drops faster than for 50/50 investors when hedge funds are introduced. As a result, the differences in the optimal allocations for different types of investors will be larger. This is also shown in table 6. Going over the latter table we see that for 33/66 investors the optimal allocations depend much more heavily on the hedge fund allocation chosen than for 50/50 investors. As a result, it will not be possible to create one single fund of hedge funds that can optimally serve 33/66 investors that plan to invest different proportions in hedge funds.

<< Insert Table 7 and 8 >>

7. A HEDGED FUND OF HEDGE FUNDS

A more direct way to create a skewness protected fund of hedge funds would be to simply have the fund purchase a put option on itself. This would protect the fund's investors against a drop in the value of the fund, and thereby sever the link between the stock market and the fund when the stock market dropped and pulled the fund with it. Of course, this does not come for free as the fund will have to pay for the put. Let's for now, however, concentrate on the risk aspects of this strategy and assume that the put is indeed available at zero costs. For different option strike price levels, table 7 shows the standard deviation, skewness and kurtosis of the overall portfolio return of a 50/50 investor who invested in such a hedged fund of hedge funds. Table 8 shows the same for a 33/66 investor. From table 7 we see that after the addition of the put the skewness of the overall portfolio return distribution still drops somewhat when the hedge fund allocation is increased but that, especially for higher strikes, the drop is limited. In addition, compared to the case without put, the standard deviation drops faster and kurtosis drops instead of rises. A similar picture emerges from table 8.

<< Insert Table 9 >>

Purely from a risk perspective, the addition of a put seems a highly beneficial move. The question remains, however, what happens to the fund's and the overall portfolio's expected return when a put is added. This depends of course on the price of the put. Although all the major derivatives houses are currently setting up desks, the market for derivatives on baskets of hedge funds is still in its infancy. It is therefore difficult to say how an option like the one we are looking for would be priced in the market. We can, however, approach the problem from another angle and investigate how much the fund could spend before its expected return would drop below its original level. This is shown in table 9, which shows that introducing the put at zero cost will raise the fund's expected return by 0.1-0.5% per month, depending on the strike price chosen. If the fund was able to purchase the desired option at these prices, it could significantly improve its risk profile while maintaining its expected return. Unfortunately, this implies a free lunch and will therefore typically not be possible. This is also clear from the bottom row of table 9, which, assuming the desired option is priced using the Black-Scholes model, shows the implied volatilities required for the above to hold. All the implieds in table 9 are substantially below our fund of hedge funds' actual volatility of 8.5%. Since it is highly unlikely that the option will be priced at an implied lower than the volatility of the underlying, this strongly suggests that the fund will have to accept a drop in expected return if it wants to protect itself with a put option.

<< Insert Table 10 >>

Since there is no liquid market for put options on baskets of hedge funds (yet) we can only guess how large the required drop in expected return would be. To get at least some idea of what to expect, however, table 10 shows the annualised loss in expected overall portfolio return due to the incorporation of a put for several (Black-Scholes) implied volatility levels and hedge fund allocations. Assuming the option is priced at 9-11% implied volatility, table 10 shows that for relatively moderate hedge fund allocations the drop in expected return is lower than 1% per annum. Although this is more than for the skewness reduction strategy discussed previously, one has to keep in mind that in this case investors also obtain a notably lower standard deviation. If this was not desired, the portfolio could be leveraged to push the standard deviation back up to its pre-put level, which would in turn also reduce the loss in expected return. Although the effect of buying a put does not seem to have too far reaching consequences for the expected return of investors' overall portfolio, it does consume a significant part of the expected fund return. With a 98.5% strike and 10% implied volatility for example, adding a put option will cost the fund 0.28% per month in expected return. On an annual basis this reduces the expected fund return from 11.9% to 8.5%.

8. CONCLUSION

In this paper we investigated whether it is possible for a fund of hedge funds to offer investors access to a diversified basket of hedge funds and provide skewness protection at the same time. We studied two alternative strategies: buying stock index puts plus leveraging and buying puts on the fund itself. We showed that in general the first strategy is too dependent on the actual asset allocation strategy followed by investors to allow a fund to be constructed that is optimal for all investors at the same time. However, for investors that invest more or less equal amounts in stocks and bonds and who keep their hedge fund allocation below 30% such a fund can indeed be structured. The second strategy does allow a fund of hedge funds to offer skewness protection to different types of investors at the same time, but compared to the optimal strategy the protection will be somewhat less accurate.

There are some other differences between both strategies as well. First, the costs, in terms of expected return foregone, of buying puts on the fund are equal to the price of those puts (plus their financing). The costs of the stock index put strategy, however, are less clear and depend on the interest rate at which the fund can leverage itself and the expected return on the asset classes involved. Second, stock index puts are a lot easier to obtain than puts in a basket of hedge funds. The lack of liquidity of the latter may drive up prices and make this route excessively expensive at times. Third, both strategies yield a significantly different risk profile. The return distribution of the hedged fund of funds (assuming a 98.5% strike and 10% implied volatility) has a skewness of 0.71. The return on the optimal 50/50 fund on the other hand has a skewness of 0.12. This large difference in skewness makes it clear that although both funds accomplish more or less the same, they do so in different ways. This in turn underlines the fact that *(funds of) hedge funds are primarily portfolio diversifiers and should therefore always be evaluated in a portfolio context, not in isolation.*

Finally, it should be noted that the idea of skewness protection conflicts with the existing ‘wisdom’ that hedge funds offer investors absolute or market neutral returns that are especially desirable in equity down markets. It will therefore be interesting to see whether any of the existing fund of funds operators will ever offer a skewness protected fund as it requires a marketing story that is quite opposite to the story that many of them have been flogging so far. On the other hand, stranger things have happened.

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Table 1: Allocations and return statistics 50/50 portfolios with hedge funds

% HF	% Stock	% Bond	Mean 1.46%	Mean 1%	SD	Skew	Kurt
0	50.0	50.0	0.95	0.72	2.49	-0.33	-0.03
5	47.5	47.5	0.95	0.73	2.43	-0.40	0.02
10	45.0	45.0	0.95	0.74	2.38	-0.46	0.08
15	42.5	42.5	0.95	0.76	2.33	-0.53	0.17
20	40.0	40.0	0.95	0.77	2.29	-0.60	0.28
25	37.5	37.5	0.96	0.78	2.25	-0.66	0.42
30	35.0	35.0	0.96	0.80	2.22	-0.72	0.58
35	32.5	32.5	0.96	0.81	2.20	-0.78	0.77
40	30.0	30.0	0.96	0.82	2.18	-0.82	0.97
45	27.5	27.5	0.96	0.84	2.17	-0.85	1.19
50	25.0	25.0	0.97	0.85	2.16	-0.87	1.41
55	22.5	22.5	0.97	0.86	2.16	-0.88	1.63
60	20.0	20.0	0.97	0.88	2.17	-0.88	1.85
65	17.5	17.5	0.97	0.89	2.18	-0.86	2.04
70	15.0	15.0	0.97	0.91	2.20	-0.82	2.22
75	12.5	12.5	0.98	0.92	2.23	-0.78	2.36
80	10.0	10.0	0.98	0.93	2.26	-0.73	2.48
85	7.5	7.5	0.98	0.95	2.30	-0.67	2.57
90	5.0	5.0	0.98	0.96	2.34	-0.60	2.63
95	2.5	2.5	0.98	0.97	2.39	-0.54	2.66
100	0.0	0.0	0.99	0.99	2.44	-0.47	2.67

Table 2: Allocations 50/50 portfolios with hedge funds and optimal skewness reduction strategy

% HF	% Stock	% Bonds	% HF	% Put	% Cash
0	50.00	50.00	0.00	0.00	0.00
5	49.34	49.34	5.19	0.12	-4.00
10	48.48	48.48	10.77	0.26	-8.00
15	47.85	47.85	16.89	0.41	-13.00
20	46.97	46.97	23.49	0.57	-18.00
25	46.22	46.22	30.81	0.74	-24.00
30	45.18	45.18	38.72	0.92	-30.00
35	43.20	43.20	46.52	1.07	-34.00
40	40.75	40.75	54.33	1.18	-37.00
45	37.62	37.62	61.55	1.21	-38.00
50	33.70	33.70	67.41	1.18	-36.00
55	29.68	29.68	72.54	1.10	-33.00
60	25.60	25.60	76.80	0.99	-29.00
65	21.72	21.72	80.68	0.88	-25.00
70	18.04	18.04	84.17	0.76	-21.00
75	14.67	14.67	88.00	0.66	-18.00
80	11.35	11.35	90.76	0.55	-14.00
85	8.37	8.37	94.82	0.45	-12.00
90	5.43	5.43	97.79	0.35	-9.00
95	2.64	2.64	100.46	0.25	-6.00
100	0.00	0.00	103.83	0.17	-4.00

Table 3: Return statistics 50/50 portfolios with hedge funds and optimal skewness reduction strategy

% HF	Mean 1.46%	Loss pa	Mean 1.00%	Loss pa	SD	Skew	Kurt
0	0.95	0.00	0.72	0.00	2.49	-0.33	-0.03
5	0.95	-0.03	0.72	-0.13	2.43	-0.33	-0.03
10	0.94	-0.08	0.72	-0.27	2.38	-0.33	-0.04
15	0.94	-0.09	0.72	-0.38	2.33	-0.33	-0.03
20	0.94	-0.12	0.73	-0.51	2.29	-0.33	-0.03
25	0.95	-0.13	0.73	-0.61	2.25	-0.33	-0.02
30	0.95	-0.16	0.74	-0.70	2.22	-0.33	0.00
35	0.94	-0.21	0.74	-0.79	2.20	-0.33	0.02

% HF	Mean 1.46%	Loss pa	Mean 1.00%	Loss pa	SD	Skew	Kurt
40	0.94	-0.25	0.75	-0.85	2.18	-0.33	0.06
45	0.94	-0.26	0.77	-0.82	2.17	-0.33	0.15
50	0.94	-0.32	0.78	-0.80	2.16	-0.33	0.28
55	0.94	-0.34	0.80	-0.74	2.16	-0.33	0.46
60	0.94	-0.35	0.82	-0.64	2.17	-0.33	0.69
65	0.94	-0.35	0.84	-0.55	2.18	-0.33	0.93
70	0.94	-0.36	0.86	-0.52	2.20	-0.33	1.18
75	0.95	-0.32	0.88	-0.44	2.23	-0.33	1.41
80	0.95	-0.33	0.90	-0.41	2.26	-0.33	1.65
85	0.96	-0.26	0.92	-0.27	2.30	-0.33	1.87
90	0.96	-0.24	0.94	-0.22	2.34	-0.33	2.06
95	0.96	-0.19	0.96	-0.20	2.39	-0.33	2.23
100	0.98	-0.12	0.98	-0.12	2.44	-0.33	2.38

Table 4: Optimal allocations fund of hedge funds portfolios (50/50 investors)

	Zero Invest		Full Invest		Zero & Full Invest		
% HF	% Stock	% Bonds	% Stocks	% Bonds	% HF	% Puts	% Cash
5	-13.19	-13.19	36.81	36.81	103.88	2.50	-80.00
10	-15.17	-15.17	34.83	34.83	107.74	2.59	-80.00
15	-14.32	-14.32	35.68	35.68	112.59	2.71	-86.67
20	-15.13	-15.13	34.87	34.87	117.43	2.83	-90.00
25	-15.12	-15.12	34.88	34.88	123.26	2.98	-96.00
30	-16.08	-16.08	33.92	33.92	129.08	3.08	-100.00

	Zero Invest		Full Invest		Zero & Full Invest		
% HF	% Stock	% Bonds	% Stocks	% Bonds	% HF	% Puts	% Cash
35	-19.42	-19.42	30.58	30.58	132.93	3.06	-97.14
40	-23.13	-23.13	26.87	26.87	135.82	2.95	-92.50
45	-27.52	-27.52	22.48	22.48	136.79	2.70	-84.44
50	-32.59	-32.59	17.41	17.41	134.82	2.37	-72.00
55	-36.95	-36.95	13.05	13.05	131.90	2.01	-60.00
60	-40.66	-40.66	9.34	9.34	128.01	1.66	-48.33
65	-43.50	-43.50	6.50	6.50	124.13	1.35	-38.46
70	-45.66	-45.66	4.34	4.34	120.24	1.09	-30.00
75	-47.11	-47.11	2.89	2.89	117.34	0.88	-24.00
80	-48.32	-48.32	1.68	1.68	113.45	0.68	-17.50
85	-48.98	-48.98	1.02	1.02	111.55	0.53	-14.12
90	-49.52	-49.52	0.48	0.48	108.65	0.39	-10.00
95	-49.85	-49.85	0.15	0.15	105.75	0.27	-6.32
100	-50.00	-50.00	0.00	0.00	103.83	0.17	-4.00

Table 5: Return statistics 50/50 portfolios with 'optimal' fund of funds

	Realized					Target				
% HF	Loss pa 1.46%	Loss pa 1%	SD	Skew	Kurt	Loss pa 1.46%	Loss pa 1%	SD	Skew	Kur
5	-0.03	-0.13	2.43	-0.33	-0.03	-0.03	-0.13	2.43	-0.33	-0.0
10	-0.05	-0.25	2.38	-0.33	-0.04	-0.08	-0.27	2.38	-0.33	-0.0
15	-0.08	-0.38	2.33	-0.33	-0.03	-0.09	-0.38	2.33	-0.33	-0.0
20	-0.11	-0.50	2.29	-0.33	-0.03	-0.12	-0.51	2.29	-0.33	-0.0

	Realized					Target				
% HF	Loss pa 1.46%	Loss pa 1%	SD	Skew	Kurt	Loss pa 1.46%	Loss pa 1%	SD	Skew	Kur
25	-0.14	-0.63	2.26	-0.33	-0.02	-0.13	-0.61	2.25	-0.33	-0.02
30	-0.16	-0.75	2.23	-0.33	-0.01	-0.16	-0.70	2.22	-0.33	0.00
35	-0.19	-0.88	2.22	-0.33	0.00	-0.21	-0.79	2.20	-0.33	0.02
40	-0.22	-1.00	2.21	-0.32	0.01	-0.25	-0.85	2.18	-0.33	0.06
45	-0.24	-1.13	2.20	-0.31	0.01	-0.26	-0.82	2.17	-0.33	0.15
50	-0.27	-1.25	2.21	-0.29	0.02	-0.32	-0.80	2.16	-0.33	0.28
55	-0.30	-1.38	2.22	-0.27	0.03	-0.34	-0.74	2.16	-0.33	0.46
60	-0.33	-1.50	2.24	-0.24	0.03	-0.35	-0.64	2.17	-0.33	0.69
65	-0.35	-1.63	2.27	-0.20	0.04	-0.35	-0.55	2.18	-0.33	0.93
70	-0.38	-1.75	2.31	-0.16	0.04	-0.36	-0.52	2.20	-0.33	1.18
75	-0.41	-1.88	2.35	-0.12	0.05	-0.32	-0.44	2.23	-0.33	1.41
80	-0.43	-2.00	2.40	-0.07	0.06	-0.33	-0.41	2.26	-0.33	1.65
85	-0.46	-2.13	2.46	-0.03	0.07	-0.26	-0.27	2.30	-0.33	1.87
90	-0.49	-2.25	2.52	0.02	0.09	-0.24	-0.22	2.34	-0.33	2.06
95	-0.52	-2.38	2.58	0.07	0.11	-0.19	-0.20	2.39	-0.33	2.23
100	-0.54	-2.50	2.66	0.12	0.13	-0.12	-0.12	2.44	-0.33	2.38

Table 6: Optimal allocations fund of hedge funds portfolios (33/66 investors)

	Zero Invest		Full Invest		Zero & Full Invest		
% HF	% Stock	% Bonds	% Stocks	% Bonds	% HF	% Puts	% Cash
5	-2.66	-5.33	30.67	61.34	104.84	3.15	-100.00
10	1.56	3.12	34.89	69.78	111.63	3.70	-120.00
15	5.05	10.11	38.39	76.77	120.32	4.52	-140.00
20	19.23	38.46	52.56	105.12	139.42	7.90	-205.00

	Zero Invest		Full Invest		Zero & Full Invest		
% HF	% Stock	% Bonds	% Stocks	% Bonds	% HF	% Puts	% Cash
25	11.58	23.16	44.91	89.83	144.91	8.35	-188.00
30	2.26	4.52	35.60	71.19	145.77	7.45	-160.00
35	-5.05	-10.10	28.29	56.57	145.69	6.60	-137.14
40	-10.49	-20.98	22.85	45.69	145.69	5.77	-120.00
45	-14.70	-29.40	18.63	37.27	145.74	5.03	-106.67
50	-18.05	-36.11	15.28	30.56	145.84	4.32	-96.00
55	-21.59	-43.18	11.74	23.49	143.06	3.53	-81.82
60	-24.59	-49.18	8.74	17.48	139.34	2.77	-68.33
65	-27.31	-54.61	6.03	12.06	133.58	2.18	-53.85
70	-29.08	-58.17	4.25	8.50	129.76	1.78	-44.29
75	-30.45	-60.91	2.88	5.76	125.91	1.46	-36.00
80	-31.50	-63.00	1.84	3.67	122.03	1.21	-28.75
85	-32.21	-64.42	1.12	2.25	119.12	1.03	-23.53
90	-32.73	-65.47	0.60	1.20	116.20	0.88	-18.89
95	-33.08	-66.17	0.25	0.50	114.29	0.75	-15.79
100	-33.33	-66.67	0.00	0.00	112.34	0.66	-13.00

Table 7: Return statistics 50/50 portfolios with hedged fund of hedge funds

	Strike 100% of Spot			Strike 99.5% of Spot		
% HF	SD	Skew	Kurt	SD	Skew	Kurt
0	2.49	-0.33	-0.03	2.49	-0.33	-0.03
5	2.40	-0.35	-0.07	2.40	-0.36	-0.07
10	2.31	-0.37	-0.12	2.32	-0.38	-0.12
15	2.22	-0.39	-0.17	2.24	-0.39	-0.17
20	2.14	-0.39	-0.23	2.16	-0.40	-0.23
25	2.06	-0.39	-0.29	2.09	-0.41	-0.30

	Strike 100% of Spot			Strike 99.5% of Spot		
% HF	SD	Skew	Kurt	SD	Skew	Kurt
30	1.98	-0.38	-0.36	2.03	-0.40	-0.37
35	1.92	-0.36	-0.42	1.97	-0.38	-0.44
40	1.85	-0.32	-0.48	1.91	-0.34	-0.50
45	1.79	-0.25	-0.53	1.86	-0.29	-0.55
50	1.74	-0.17	-0.56	1.82	-0.21	-0.59
	Strike 99% of Spot			**Strike 98.5% of Spot**		
% HF	**SD**	**Skew**	**Kurt**	**SD**	**Skew**	**Kurt**
5	2.41	-0.36	-0.07	2.41	-0.36	-0.07
10	2.33	-0.38	-0.12	2.34	-0.39	-0.12
15	2.26	-0.40	-0.18	2.27	-0.41	-0.18
20	2.19	-0.42	-0.24	2.21	-0.43	-0.24
25	2.13	-0.43	-0.31	2.15	-0.44	-0.30
30	2.07	-0.42	-0.38	2.10	-0.45	-0.36
35	2.01	-0.41	-0.45	2.05	-0.44	-0.43
40	1.97	-0.38	-0.51	2.01	-0.41	-0.49
45	1.92	-0.33	-0.57	1.97	-0.38	-0.54
50	1.89	-0.27	-0.61	1.94	-0.32	-0.58
	Strike 98% of Spot			**Strike 97.5% of Spot**		
% HF	**SD**	**Skew**	**Kurt**	**SD**	**Skew**	**Kurt**
5	2.42	-0.37	-0.07	2.42	-0.37	-0.06
10	2.35	-0.40	-0.11	2.35	-0.40	-0.10
15	2.28	-0.42	-0.16	2.29	-0.44	-0.14
20	2.22	-0.45	-0.22	2.24	-0.46	-0.18
25	2.17	-0.46	-0.27	2.18	-0.49	-0.23
30	2.12	-0.47	-0.33	2.14	-0.50	-0.27
35	2.08	-0.47	-0.38	2.10	-0.50	-0.31
40	2.04	-0.45	-0.44	2.06	-0.49	-0.36
45	2.01	-0.42	-0.48	2.03	-0.46	-0.39
50	1.98	-0.37	-0.52	2.01	-0.42	-0.42

Table 8: Return statistics 33/66 portfolios with hedged fund of hedge funds

	Strike 100% of Spot			Strike 99.5% of Spot		
% HF	SD	Skew	Kurt	SD	Skew	Kurt
0	2.01	0.03	0.21	2.01	0.03	0.21
5	1.94	-0.01	0.14	1.94	-0.02	0.13
10	1.87	-0.06	0.05	1.88	-0.06	0.05
15	1.80	-0.10	-0.04	1.82	-0.11	-0.05

	Strike 100% of Spot			Strike 99.5% of Spot		
% HF	SD	Skew	Kurt	SD	Skew	Kurt
20	1.73	-0.13	-0.14	1.76	-0.15	-0.16
25	1.68	-0.16	-0.25	1.71	-0.18	-0.27
30	1.62	-0.17	-0.36	1.67	-0.20	-0.39
35	1.58	-0.17	-0.47	1.63	-0.20	-0.49
40	1.54	-0.14	-0.55	1.60	-0.18	-0.58
45	1.51	-0.08	-0.61	1.58	-0.13	-0.65
50	1.49	0.00	-0.62	1.56	-0.06	-0.67
	Strike 99% of Spot			**Strike 98.5% of Spot**		
% HF	**SD**	**Skew**	**Kurt**	**SD**	**Skew**	**Kurt**
5	1.95	-0.02	0.13	1.95	-0.03	0.12
10	1.89	-0.07	0.03	1.90	-0.08	0.03
15	1.83	-0.13	-0.07	1.85	-0.14	-0.07
20	1.78	-0.17	-0.18	1.80	-0.19	-0.18
25	1.74	-0.21	-0.29	1.77	-0.23	-0.29
30	1.70	-0.23	-0.40	1.74	-0.26	-0.40
35	1.67	-0.24	-0.51	1.71	-0.28	-0.50
40	1.65	-0.23	-0.59	1.69	-0.27	-0.58
45	1.64	-0.20	-0.66	1.68	-0.25	-0.64
50	1.63	-0.14	-0.69	1.68	-0.20	-0.67
	Strike 98% of Spot			**Strike 97.5% of Spot**		
% HF	**SD**	**Skew**	**Kurt**	**SD**	**Skew**	**Kurt**
5	1.96	-0.03	0.12	1.96	-0.03	0.13
10	1.90	-0.09	0.03	1.91	-0.10	0.04
15	1.86	-0.15	-0.07	1.87	-0.17	-0.05
20	1.82	-0.21	-0.17	1.83	-0.23	-0.15
25	1.78	-0.26	-0.28	1.80	-0.28	-0.24
30	1.76	-0.29	-0.38	1.77	-0.32	-0.33
35	1.74	-0.31	-0.47	1.75	-0.35	-0.40
40	1.72	-0.32	-0.54	1.74	-0.36	-0.47
45	1.72	-0.30	-0.59	1.74	-0.34	-0.51
50	1.72	-0.26	-0.62	1.75	-0.31	-0.53

Table 9: Gains in expected fund return from incorporating put at zero costs

	Strike as % of Spot					
	100	**99.5**	**99.0**	**98.5**	**98.0**	**97.5**
Gain	0.50	0.36	0.25	0.18	0.13	0.10
Imp. Vol.	5.65	6.05	6.34	6.73	7.11	7.60

Table 10: Annualised loss in expected overall portfolio return due to incorporating put in fund of hedge funds

	9% Vol.	**10% Vol.**	**11% Vol.**		**9% Vol.**	**10% Vol.**	**11% Vol.**
Strike	**5% HF**			**Strike**	**10% HF**		
100	-0.23	-0.30	-0.37	100	-0.46	-0.61	-0.74

	9% Vol.	10% Vol.	11% Vol.		9% Vol.	10% Vol.	11% Vol.
99.5	-0.19	-0.26	-0.33	99.5	-0.39	-0.52	-0.65
99.0	-0.15	-0.21	-0.27	99.0	-0.31	-0.43	-0.55
98.5	-0.12	-0.17	-0.23	98.5	-0.23	-0.34	-0.46
98.0	-0.08	-0.13	-0.18	98.0	-0.16	-0.25	-0.36
97.5	-0.05	-0.09	-0.13	97.5	-0.10	-0.18	-0.26
Strike	**15% HF**			**Strike**	**20% HF**		
100	-0.69	-0.91	-1.11	100	-0.92	-1.21	-1.48
99.5	-0.58	-0.78	-0.98	99.5	-0.78	-1.04	-1.30
99.0	-0.46	-0.64	-0.82	99.0	-0.62	-0.86	-1.10
98.5	-0.35	-0.51	-0.69	98.5	-0.47	-0.68	-0.92
98.0	-0.24	-0.38	-0.54	98.0	-0.31	-0.51	-0.72
97.5	-0.14	-0.27	-0.40	97.5	-0.19	-0.36	-0.53
Strike	**25% HF**			**Strike**	**30% HF**		
100	-1.16	-1.52	-1.85	100	-1.39	-1.82	-2.21
99.5	-0.97	-1.30	-1.63	99.5	-1.16	-1.56	-1.96
99.0	-0.77	-1.07	-1.37	99.0	-0.93	-1.29	-1.65
98.5	-0.58	-0.85	-1.15	98.5	-0.70	-1.02	-1.38
98.0	-0.39	-0.63	-0.90	98.0	-0.47	-0.76	-1.08
97.5	-0.24	-0.45	-0.66	97.5	-0.29	-0.54	-0.79
Strike	**35% HF**			**Strike**	**40% HF**		
100	-1.62	-2.12	-2.58	100	-1.85	-2.42	-2.95
99.5	-1.36	-1.82	-2.28	99.5	-1.55	-2.08	-2.61
99.0	-1.08	-1.50	-1.92	99.0	-1.24	-1.72	-2.20
98.5	-0.81	-1.19	-1.61	98.5	-0.93	-1.36	-1.84
98.0	-0.55	-0.89	-1.26	98.0	-0.63	-1.01	-1.45
97.5	-0.34	-0.63	-0.92	97.5	-0.38	-0.72	-1.06
Strike	**45% HF**			**Strike**	**50% HF**		
100	-2.08	-2.73	-3.32	100	-2.31	-3.03	-3.69
99.5	-1.75	-2.34	-2.93	99.5	-1.94	-2.60	-3.26
99.0	-1.39	-1.93	-2.47	99.0	-1.55	-2.15	-2.75
98.5	-1.05	-1.53	-2.07	98.5	-1.16	-1.70	-2.30
98.0	-0.71	-1.14	-1.63	98.0	-0.79	-1.27	-1.81
97.5	-0.43	-0.81	-1.19	97.5	-0.48	-0.90	-1.32