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Taking the Sting Out of Hedge Funds

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Abstract

Although the inclusion of hedge funds in an investment portfolio can significantly improve that portfolio's mean-variance characteristics, it can also be expected to lead to significantly lower skewness and higher kurtosis. In this paper we show how this highly undesirable side-effect can be neutralized by allocating a fraction of wealth to out-of-the-money put options on the relevant equity index. Based on monthly return data over the period 1994-2001 we show that investors who want to fully eradicate the negative skewness of portfolios containing stocks, bonds and hedge funds will have to sacrifice a not insignificant part of their expected return. Investors who limit themselves to neutralizing only the additional skewness caused by the inclusion of hedge funds will be able to do so at much more favourable terms, however. The latter only need to allocate a small fraction of wealth to index puts and accept a drop in expected return that is unlikely to exceed 1% per annum, depending on the hedge fund allocation. This means that in the current low interest rate environment the costs of eliminating the unwanted skewness effect of hedge funds need not be prohibitively high.

1. INTRODUCTION

Due to their relatively weak correlation with other asset classes, hedge funds can play an important role in risk reduction and yield enhancement strategies. Recent research, however, has shown that this diversification service does not come for free. As shown in Amin and Kat (2002b, 2003), although the inclusion of hedge funds in a portfolio may significantly improve that portfolio's mean-variance characteristics, it can also be expected to lead to significantly lower skewness as well as higher kurtosis. This means that the case for hedge funds is less straightforward than often suggested and includes a definite trade-off between profit and loss potential.

The most important side-effect of including hedge funds in an investment portfolio is an increase in the portfolio return distribution's skewness. The sting of hedge funds is literally in the tail as in terms of skewness, hedge funds and equity do not mix very well. When things go wrong in the stock market, they also tend to go wrong for hedge funds as a significant drop in stock prices will often be accompanied by a drop in market liquidity, a widening of a multitude of spreads, etc. Equity market neutral funds for example have a tendency to be long in smaller stocks and short in larger stocks and need liquidity to maintain market neutrality. As a result, when the stock market comes down this type of funds can be expected to have a hard time. Likewise, when the stock market comes down many mergers and acquisitions are likely to be postponed which will have a negative impact on the performance of risk arbitrage funds. Problems are not limited to funds that invest in equity though. A drop in stock prices will often also lead to a widening of credit spreads, which in turn will seriously damage the performance of fixed income hedge funds. As they all share it, diversification among different funds will not mitigate this problem.

Since for most investors the increase in negative skewness that tends to come with hedge fund investing is highly undesirable, it is important to look for ways to neutralize this effect. One solution is to buy hedge funds in 'guaranteed' form only. In essence, this means buying a put on one's hedge fund portfolio so that in down markets the link between the hedge fund portfolio and the equity market is severed. Unfortunately, the market for puts on baskets of hedge funds is still in an early stage. As a result, counterparties for the required contracts are likely to be hard to find as well as expensive. With hedge funds so closely related to the ups and especially downs of the equity market, there is a very simple alternative though: the purchase of out-of-the-money puts on the equity index. In this paper we study this strategy in more detail.

2. FOUR ASSET CLASSES

We distinguish four different asset classes: stocks, bonds, hedge funds and out-of-the-money puts. To represent stocks we use the S&P 500 index, while bonds are represented by the 10-year Salomon Brothers Government Bond index. Since nowadays most investors invest in a basket of hedge funds instead of a single fund, to represent hedge funds we use an equally-weighted portfolio of 20 different funds. Hedge fund return data were obtained from Tremont TASS, which is one of the largest hedge fund databases currently available to academic researchers. In a recent paper, Amin and Kat (2002a) showed that concentrating on surviving funds will not only lead investors to overestimate the mean return on individual hedge funds by around 2%, it will also introduce a significant downward bias in estimates of the standard deviation, an upward bias in the skewness and a downward bias in the kurtosis estimates of individual fund returns. To correct for this we created 455 7-year monthly return series by, starting off with the 455 funds that were alive (in the TASS database) in June 1994, replacing every fund that closed down during the sample period by a fund randomly selected from the set of funds alive at the time of closure, following the same type of strategy and of similar age and size. Next, we created 500 different equally-weighted portfolios containing 20 hedge funds each by random sampling without replacement from the above 455 series. From the monthly returns on these portfolios we calculated the mean, standard deviation, skewness, and kurtosis. Subsequently, we determined the median value of each of these statistics and selected the portfolio whose sample statistics came closest to the latter median values. We use this portfolio, i.e. the median portfolio of 20 individual hedge funds, to represent hedge funds.

Apart from stocks, bonds and hedge funds we distinguish a fourth asset class: out-of-the-money put options on the S&P 500. The returns on this asset class are taken to be the returns on the following rollover strategy. On the first trading day of the month we buy a put on the S&P 500 that expires the next month. On the first trading day of the next month we sell the option and replace it with another S&P 500 put that expires a month later, etc. Since we are primarily interested in the tail of the return distribution, we do not buy at-the-money options, i.e. options whose present value of strike is closest to the index value, but the next lower strike. With S&P 500 options trading in strike increments of five index points, this means that for the options we buy the ratio of index value to present value of the strike price is 1.01. We could of course buy options that are further out-of-the-money but then we might run into liquidity problems. For simplicity, we assume that the above strategy can be executed without costs. Implicitly, the same assumption is made for the other asset classes.

<< Insert Table 1 >>

Table 1 shows the basic statistics of the above four asset classes' return distributions estimated over the period June 1994 – May 2001. From the table we see that the returns generated by the four asset classes are very different. The S&P 500 offers a relatively high mean but also a relatively high standard deviation and substantial negative skewness. Bonds show the opposite pattern. The relatively low mean comes with a low standard deviation and positive skewness. The hedge fund portfolio lives up to its reputation in the sense that it offers a mean that is similar to that of stocks but a standard deviation that is more like that of bonds. This does not come for free though. The returns on the hedge fund portfolio exhibit negative skewness and relatively high kurtosis. Not surprisingly, with a standard deviation of a whopping 94.56% the returns on the put strategy are of a very extreme nature. The strategy's returns also exhibit very high positive skewness, which is a very valuable characteristic. The negative mean therefore does not come as a complete surprise.

3. TAKING THE SKEWNESS OUT OF THE S&P 500

We start with a simple example. Over the period June 1994 – May 2001 monthly S&P 500 index returns exhibit a skewness of –0.82. Over the same period, our strategy of rolling S&P 500 puts generated returns with a positive skewness of 2.17. It therefore makes sense to use the option strategy to neutralize the negative skewness in the index. Since its return behaviour is so extreme, we do not have to invest a lot in the option strategy to do so. An allocation of only 1.95% will be enough. The statistics of the resulting return distribution can be found in the second column of table 2. From the table we see that the return on a portfolio of 98.05% S&P 500 and 1.95% puts will have a mean of 1.03%, a standard deviation of 2.81%, a skewness of 0.00 and an excess kurtosis of –0.24.

<< Insert Table 2 >>

Table 2 shows that investing only a small fraction of one's wealth in puts will significantly change the shape of the return distribution. Apart from the complete disappearance of skewness, the mean return drops by 0.43%, the standard deviation drops by 1.58% and the kurtosis drops by 1.30. We can restore the standard deviation and most of the mean by leveraging the portfolio, which can easily be done by purchasing S&P 500 futures for example. Suppose we levered the portfolio by a factor 1.57, i.e. we borrowed 57% of the portfolio value and invested the proceeds in the original portfolio. Assuming we can borrow at 4% per annum, this would give us a return distribution with characteristics as in the third column of table 2. Skewness and kurtosis would of course remain unchanged. The standard deviation on the other hand would return to its former level of 4.39% and, since the unlevered portfolio's mean far exceeds the interest rate of 4%, the mean would rise to 1.42%, which is only 0.04% lower than that of the S&P 500 itself. In sum, this example shows that at a cost (in terms of expected return foregone) of only 0.5% per annum we are able to strip the S&P 500 return distribution from all its negative skewness while maintaining its standard deviation. In the process we also eliminate much of the excess kurtosis in S&P 500 returns.

4. TAKING THE SKEWNESS OUT OF HEDGE FUNDS

We repeated the same routine for the hedge fund portfolio. The results are shown in the last two columns of table 2. By itself, the hedge fund portfolio return has a skewness of –0.47. To take this back to zero we need to invest 0.54% in the rolling put strategy. As shown in table 2, apart from zero skewness, the returns on a portfolio of 99.46% hedge funds and 0.54% puts will have a mean of 0.87%, a standard deviation of 2.17% and an excess kurtosis of 1.71. If we were to leverage this portfolio by a factor 1.13, again assuming we can borrow at 4%, the resulting portfolio's return distribution would have a mean of 0.94%, a standard deviation equal to that of the original hedge fund portfolio, zero skewness and an excess kurtosis of 1.71. In short, it would take a reduction in expected return of 0.6% per annum to strip the hedge fund portfolio from its negative skewness while maintaining its standard deviation. As a bonus, we would reduce kurtosis by 0.96.

It is interesting to note that in terms of expected return foregone it turns out to be more expensive to eliminate the –0.47 skewness of hedge funds than the –0.82 skewness of the S&P 500. The reason for this is that the returns on the hedge fund portfolio are substantially less volatile than those on the S&P 500 index. The hedge fund portfolio therefore requires less leveraging and as a result picks up less additional expected return. We will encounter this phenomenon again later in the paper.

5. S&P 500 PLUS HEDGE FUNDS

Since, in popular terms, hedge funds offer equity returns with bond risk, some investors use hedge funds to reduce the risk of their equity portfolio without having to give up much expected return. However, when hedge funds and equity are combined skewness drops further than one might expect because in equity down markets the relationship between hedge funds and equity grows stronger. This means that if we want to take the skewness out of a portfolio made up of stocks and hedge funds, the optimal allocation to the rolling put strategy will depend on the size of the hedge fund allocation. We repeated the procedure used earlier for hedge fund allocations ranging between 0% and 100% in 5% steps. The results are shown in table 3 and 4 and figure 1 and 2.

<< Insert Table 3 and figure 1>>

The first five columns of table 3 show the return statistics of the various combinations of stocks and hedge funds without puts or leverage. Starting with 100% invested in stocks, we see that if the hedge fund allocation increases, the skewness of the portfolio return drops while the kurtosis goes up. Although with –0.47 the skewness of the hedge fund returns is substantially higher than that of the S&P 500, when 55% of the portfolio value is allocated to hedge funds the skewness of the portfolio return drops to a very significant –1.07. To eliminate the skewness in the return distributions of the above portfolios we may again purchase some out-of-the-money S&P 500 puts. The required allocations are shown in the sixth column of table 3, followed by the resulting means, standard deviations and kurtosis values. As before, it appears we do not need to allocate too much to puts to eliminate these portfolios' skewness. The required allocation is never higher than 2%. It is interesting to note that there does not seem to be a clear-cut relationship between the degree of skewness to be eliminated and the required allocation to puts, which underlines the complexity of the relationship between the three asset classes involved.

<< Insert Table 4 and Figure 2 >>

From figure 1, which shows the change in the mean, standard deviation and kurtosis of the portfolio return, it is clear that, apart from eliminating all skewness, the purchase of puts also has a strong impact on the mean, standard deviation and kurtosis of the portfolio return. All three statistics are substantially lower than in the case without puts. To bring at least the standard deviation back to its initial level we can leverage the portfolio again. The required degree of leverage is shown in the third column of table 4 and varies between 1.13 and 1.77, depending on the hedge fund allocation. From the table we also see that, still under the assumption that we can borrow at 4%, leveraging the portfolio not only restores the standard deviations but to a large extend also restores the means. The changes in the mean and kurtosis of the portfolio return are shown in figure 2. Concentrating on the lower hedge fund allocations, we see that the loss in expected return due to the elimination of skewness amounts to not more than 0.5% per annum and is accompanied by a significant drop in kurtosis as well.

6. S&P 500 PLUS BONDS PLUS HEDGE FUNDS

Most investors will fit hedge funds in with their existing portfolio of stocks and bonds. We therefore repeated the previous exercise with portfolios of stocks, bonds and hedge funds as well, always assuming an equal allocation to stocks and bonds. The results can be found in table 5 and 6 and figure 3 and 4.

<< Insert Table 5 and figure 3 >>

From the first five columns of table 5 we see that the return on a portfolio of 50% stocks and 50% bonds has a mean of 0.96%, a standard deviation of 2.49, a skewness of –0.33 and a kurtosis of –0.03. When hedge funds are introduced, the skewness of the portfolio return drops substantially. With 55% invested in hedge funds the skewness of the portfolio return is a marked –0.88. The allocations to S&P 500 puts required to eliminate all skewness can be found in the sixth column of table 5, which shows that all allocations are well below 2% and many even below 1%. The resulting values for the mean, standard deviation and kurtosis can be found in the last three columns, while the changes relative to the case without puts are displayed in figure 3. From the latter we see that although the put allocations are only small, they again have quite an impact on the return distributions. Apart from eliminating all skewness, they significantly reduce the mean, standard deviation and kurtosis of the portfolio return.

<< Insert Table 6 and figure 4 >>

Leveraging the above portfolios to bring the standard deviations back to their original values would yield the results shown in table 6 and figure 4. Table 6 shows that to eliminate the negative skewness caused by the particular dependence structure between stocks and hedge funds investors will have to sacrifice between 0.4% and 2% per annum in expected return. Despite the lower levels of skewness to be eliminated, this is substantially more than in the previous case without bonds. This is due to the fact that portfolios with bonds are substantially less volatile than portfolios without. As a result, they require less leveraging and therefore pick up less additional expected return.

<< Insert Figure 5 and 6 >>

The process of skewness reduction is graphically illustrated in figure 5 and 6. Based on monthly return data over the period June 1994 – May 2001, figure 5 shows the frequency distribution of the returns on a portfolio of 40% stocks, 40% bonds and 20% hedge funds. The negative skewness of –0.60 shows up clearly in the form of a relatively long tail to the left. Figure 6 shows the frequency distribution of the returns on the portfolio resulting from our skewness elimination procedure, i.e. a portfolio of 60% stocks, 60% bonds, 30% hedge funds, 2% puts and -52% cash. The latter distribution has the same standard deviation and almost the same kurtosis as the distribution shown in figure 5 but, as shows clearly from the graph, it is no longer skewed.

<< Insert Table 7 and Figure 7 >>

There are two potential problems with the above. First, to get rid of all skewness investors will have to give up a significant slice of expected return. Second, the required degree of leverage is quite high, which can present a problem for the hedge fund part of the portfolio. We should therefore reconsider whether we really want to eliminate all skewness or only neutralize the additional skewness that results from the inclusion of hedge funds. Since investors seemed happy with the –0.33 skewness of the initial 50% stock and 50% bond portfolio, one might argue that the target skewness should not be 0.00 but a much less ambitious –0.33 instead. Repeating the above procedure aiming for a skewness level of –0.33 instead of 0.00 we obtain the results reported in table 7 and figure 7. From table 7 we see that in this case the required allocation to the put strategy as well as the required degree of leverage are much lower than before. Even more important, the costs of eliminating only the skewness resulting from the inclusion of hedge funds appear to be very modest: between 0.05% and 0.35% per annum.

7. SENSITIVITY ANALYSIS

Of course, the previous conclusions heavily depend on the assumption that investors can leverage their portfolios at 4%, which in the current low interest rate environment does not seem unrealistic. If the interest rate were higher, the costs of the skewness reduction strategy would be higher as well because the difference between the expected return on the unlevered portfolio and the interest rate would be smaller. To get an indication of the seriousness of this effect we repeated the last case assuming an interest rate of 6%. The results can be found in the fourth column of table 8 and are graphically shown in figure 7. From the latter we clearly see that if the interest rate were 6% instead of 4%, the costs of the strategy in terms of expected return foregone would rise very substantially. However, the costs would still not be higher than 1% per annum.

<< Insert Table 8 >>

Implicitly, our previous conclusions are also based on the assumption that the expected return on stocks is equal to the average monthly S&P 500 return over the period June 1994 –May 2001. Since the latter period includes one of the biggest bull markets ever, this may seriously overestimate investors' current view on the stock market, especially after three years of falling stock prices worldwide. If the expected return on stocks were lower, the costs of the skewness reduction strategy would be higher because the difference between the expected return on the unlevered portfolio and the interest rate would again be smaller. The third column of table 8 shows what would happen if the expected monthly return on stocks were equal to 1% instead of 1.46% (for simplicity keeping the expected return on the other three asset classes unchanged). From the table we see that, similar to the case of a higher interest rate, a lower expected stock return would substantially raise the costs of the skewness reduction strategy. In none of the cases studied do the costs exceed 0.85% per annum, however.

8. THE COSTS OF DIVERSIFICATION WITH HEDGE FUNDS

When we target a skewness level equal to that before the inclusion of hedge funds in the portfolio, we can interpret the loss in expected return resulting from the skewness reduction strategy as the costs of the reduction in portfolio return standard deviation due to the introduction of hedge funds. Put another way, the skewness reduction strategy allows us to put a price tag on the diversification benefits of hedge funds. For various hedge fund allocations, the last column in table 8 shows the improvement in standard deviation resulting from the inclusion of hedge funds in a portfolio of 50% stocks and 50% bonds, assuming that the proportions of wealth invested in stocks and bonds are always equal. From table 8 it is easily calculated that the loss in expected return per unit of reduction in standard deviation varies between 0.5% and 3% per annum, depending on the hedge fund allocation, the interest rate and the expected stock return. This again makes it very clear that the diversification benefits of hedge funds are not a free lunch but are bought by taking on more negative skewness.

9. CONCLUSION

Although the inclusion of hedge funds in an investment portfolio can significantly improve that portfolio's mean-variance characteristics, it can also be expected to lead to significantly lower skewness and higher kurtosis. We showed how this highly undesirable side-effect can be neutralized by allocating a fraction of wealth to out-of-the-money put options on the relevant equity index. Based on monthly return data over the period 1994-2001 we show that investors who want to fully eradicate the negative skewness of portfolios containing stocks, bonds and hedge funds will have to sacrifice a not insignificant part of their expected return. Investors who limit themselves to neutralizing only the additional skewness caused by the inclusion of hedge funds will be able to do so at much more favourable terms, however. The latter only need to allocate a small fraction of wealth to index puts and accept a drop in expected return that is unlikely to exceed 1% per annum, depending on the hedge fund allocation. This means that in the current low interest rate environment the costs of eliminating the unwanted skewness effect of hedge funds need not be prohibitively high.

REFERENCES

Amin, G. and H. Kat (2002a), Welcome to the Dark Side: Hedge Fund Attrition and Survivorship Bias 1994 – 2001, Working Paper ISMA Centre, University of Reading.

Amin, G. and H. Kat (2002b), Diversification and Yield Enhancement with Hedge Funds, forthcoming *Journal of Alternative Investments*, Winter.

Amin, G. and H. Kat (2003), Stocks, Bonds and Hedge Funds: No Free Lunch!, forthcoming *Journal of Portfolio Management,* Summer.

Table 1: Basic statistics S&P 500, bonds, hedge funds and OTM S&P 500 puts

	S&P 500	Bonds	HF	OTM Put
Mean	1.46	0.45	0.99	-20.58
Std. Deviation	4.39	1.77	2.44	94.56
Skewness	-0.82	0.58	-0.47	2.17
Excess Kurt.	1.05	1.45	2.67	5.60

Table 2: Effects of combining stocks and hedge funds with puts and leverage

	S&P 500 +		HF +	
	1.95% in Puts	57% Leverage	0.54% in Puts	13% Leverage
Mean	1.03	1.42	0.87	0.94
Std. Deviation	2.81	4.39	2.17	2.44
Skewness	0.00	0.00	0.00	0.00
Excess Kurt.	-0.24	-0.24	1.71	1.71
Change Mean	-0.43	-0.04	-0.12	-0.05
Change St. Dev.	-1.58	0.00	-0.27	0.00
Change Skew	+0.82	+0.82	+0.47	+0.47
Change Kurt	-1.30	-1.30	-0.96	-0.96

Table 3: Effects of combining portfolios of stocks and hedge funds with puts

	S&P 500 + HF					S&P 500 + HF + Put		
% HF	Mean	SD	Skew	Kurt	% Put	Mean	SD	Kurt
0	1.46	4.39	-0.82	1.05	1.95	1.03	2.81	-0.25
5	1.44	4.25	-0.84	1.14	1.96	1.01	2.67	-0.25
10	1.41	4.11	-0.87	1.24	1.96	0.98	2.54	-0.25
15	1.39	3.98	-0.90	1.35	1.98	0.96	2.41	-0.25
20	1.37	3.84	-0.93	1.46	1.98	0.94	2.28	-0.26
25	1.34	3.71	-0.96	1.59	2.00	0.90	2.16	-0.26
30	1.32	3.58	-0.98	1.72	2.00	0.88	2.05	-0.25
35	1.29	3.46	-1.01	1.86	2.00	0.85	1.96	-0.25
40	1.27	3.34	-1.03	2.01	1.98	0.84	1.89	-0.21
45	1.25	3.23	-1.05	2.17	1.94	0.83	1.84	-0.15
50	1.22	3.12	-1.06	2.32	1.88	0.81	1.80	-0.07
55	1.20	3.01	-1.07	2.47	1.80	0.81	1.78	0.05
60	1.18	2.91	-1.06	2.62	1.70	0.81	1.78	0.21
65	1.15	2.82	-1.04	2.74	1.58	0.81	1.79	0.39
70	1.13	2.74	-1.01	2.85	1.45	0.82	1.82	0.59
75	1.11	2.67	-0.97	2.92	1.31	0.83	1.87	0.79
80	1.08	2.60	-0.90	2.96	1.16	0.83	1.91	1.01
85	1.06	2.54	-0.82	2.96	1.01	0.84	1.96	1.21
90	1.03	2.50	-0.71	2.90	0.86	0.84	2.03	1.38
95	1.01	2.47	-0.60	2.81	0.70	0.86	2.10	1.56
100	0.99	2.44	-0.47	2.67	0.54	0.87	2.17	1.71

Table 4: Effects of combining portfolios of stocks and hedge funds with puts and leverage

			S&P 500 + HF + Put		
% HF	**% Put**	**Leverage**	**Mean**	**Change Mean p.a.**	**Change Kurt**
0	1.95	1.56	1.42	-0.47	-1.30
5	1.96	1.59	1.41	-0.42	-1.39
10	1.96	1.62	1.38	-0.34	-1.49
15	1.98	1.66	1.37	-0.30	-1.60
20	1.98	1.68	1.34	-0.33	-1.72
25	2.00	1.72	1.31	-0.33	-1.85
30	2.00	1.74	1.29	-0.39	-1.97
35	2.00	1.76	1.25	-0.47	-2.11
40	1.98	1.77	1.23	-0.52	-2.22
45	1.94	1.76	1.20	-0.60	-2.32
50	1.88	1.74	1.17	-0.65	-2.39
55	1.80	1.69	1.14	-0.77	-2.42
60	1.70	1.63	1.11	-0.85	-2.41
65	1.58	1.57	1.08	-0.86	-2.35
70	1.45	1.50	1.06	-0.90	-2.26
75	1.31	1.44	1.04	-0.88	-2.13
80	1.16	1.36	1.01	-0.87	-1.95
85	1.01	1.30	0.99	-0.82	-1.75
90	0.86	1.23	0.97	-0.77	-1.52
95	0.70	1.18	0.95	-0.68	-1.25
100	0.54	1.13	0.94	-0.56	-0.96

Table 5: Effects of combining portfolios of stocks, bonds and hedge funds with puts

	S&P 500 + Bonds + HF					S&P 500 + Bonds + HF + Put		
% HF	Mean	SD	Skew	Kurt	% Put	Mean	SD	Kurt
0	0.95	2.49	-0.33	-0.03	0.61	0.82	2.05	0.02
5	0.95	2.43	-0.40	0.02	0.72	0.80	1.92	0.04
10	0.95	2.38	-0.46	0.08	0.85	0.77	1.78	0.07
15	0.95	2.33	-0.53	0.17	1.01	0.73	1.64	0.13
20	0.95	2.29	-0.60	0.28	1.21	0.69	1.50	0.20
25	0.96	2.25	-0.66	0.42	1.5	0.64	1.35	0.23
30	0.96	2.22	-0.72	0.58	1.68	0.60	1.30	0.10
35	0.96	2.20	-0.78	0.77	1.70	0.59	1.29	0.00
40	0.96	2.18	-0.82	0.97	1.68	0.60	1.29	-0.07
45	0.96	2.17	-0.85	1.19	1.60	0.62	1.32	-0.09
50	0.97	2.16	-0.87	1.41	1.50	0.65	1.35	-0.06
55	0.97	2.16	-0.88	1.63	1.38	0.67	1.40	0.03
60	0.97	2.17	-0.88	1.85	1.26	0.70	1.47	0.19
65	0.97	2.18	-0.86	2.04	1.14	0.72	1.54	0.39
70	0.97	2.20	-0.82	2.22	1.04	0.75	1.62	0.61
75	0.98	2.23	-0.78	2.36	0.94	0.78	1.71	0.83
80	0.98	2.26	-0.73	2.48	0.86	0.80	1.79	1.04
85	0.98	2.30	-0.67	2.57	0.77	0.81	1.89	1.24
90	0.98	2.34	-0.60	2.63	0.69	0.83	1.98	1.42
95	0.98	2.39	-0.54	2.66	0.62	0.85	2.07	1.56
100	0.99	2.44	-0.47	2.67	0.54	0.87	2.17	1.71

Table 6: Effects of combining portfolios of stocks, bonds and hedge funds with puts and leverage

			S&P 500 + Bond + HF + Put		
% HF	**% Put**	**Leverage**	**Mean**	**Change Mean p.a.**	**Change Kurt**
0	0.61	1.21	0.92	-0.36	0.05
5	0.72	1.27	0.92	-0.37	0.02
10	0.85	1.33	0.91	-0.48	-0.01
15	1.01	1.42	0.90	-0.59	-0.04
20	1.21	1.52	0.88	-0.88	-0.08
25	1.50	1.65	0.83	-1.54	-0.19
30	1.68	1.71	0.79	-2.08	-0.48
35	1.70	1.71	0.78	-2.20	-0.77
40	1.68	1.69	0.78	-2.13	-1.04
45	1.60	1.65	0.80	-1.90	-1.28
50	1.50	1.60	0.83	-1.65	-1.47
55	1.38	1.54	0.86	-1.38	-1.60
60	1.26	1.48	0.87	-1.15	-1.66
65	1.14	1.41	0.89	-1.01	-1.65
70	1.04	1.35	0.89	-0.94	-1.61
75	0.94	1.30	0.91	-0.84	-1.53
80	0.86	1.26	0.91	-0.79	-1.44
85	0.77	1.22	0.92	-0.72	-1.33
90	0.69	1.18	0.92	-0.70	-1.21
95	0.62	1.15	0.92	-0.67	-1.10
100	0.54	1.13	0.94	-0.56	-0.96

Table 7: Effects of combining portfolios of stocks, bonds and hedge funds with puts and leverage with a skewness target of –0.33 instead of 0.00

			S&P 500 + Bond + HF + Put			
% HF	% Put	Leverage	Mean	Change Mean p.a.	Kurt	Change Kurt
0	0.00	1.00	0.95	0.00	-0.03	0.00
5	0.12	1.04	0.95	-0.03	-0.03	-0.05
10	0.24	1.08	0.94	-0.08	-0.04	-0.12
15	0.36	1.13	0.94	-0.09	-0.03	-0.20
20	0.48	1.18	0.94	-0.12	-0.03	-0.31
25	0.60	1.24	0.95	-0.13	-0.02	-0.44
30	0.71	1.30	0.95	-0.16	0.00	-0.58
35	0.80	1.34	0.94	-0.21	0.02	-0.75
40	0.86	1.37	0.94	-0.25	0.06	-0.91
45	0.88	1.38	0.94	-0.26	0.15	-1.04
50	0.87	1.36	0.94	-0.32	0.28	-1.13
55	0.83	1.33	0.94	-0.34	0.46	-1.17
60	0.77	1.29	0.94	-0.35	0.69	-1.16
65	0.70	1.25	0.94	-0.35	0.93	-1.11
70	0.63	1.21	0.94	-0.36	1.18	-1.04
75	0.56	1.18	0.95	-0.32	1.41	-0.95
80	0.48	1.14	0.95	-0.33	1.65	-0.83
85	0.40	1.12	0.96	-0.26	1.87	-0.70
90	0.32	1.09	0.96	-0.24	2.06	-0.57
95	0.24	1.06	0.96	-0.19	2.23	-0.43
100	0.16	1.04	0.98	-0.12	2.38	-0.29

Table 8: Effects different assumptions regarding interest rates and expected return on stocks with a skewness target of –0.33

	4% Interest		6% Interest	
	Drift 1.46%	Drift 1.00%	Drift 1.46%	
% HF	Change Mean p.a	Change Mean p.a	Change Mean p.a	Change in SD
0	0.00	0.00	0.00	0.00
5	-0.03	-0.13	-0.11	-0.06
10	-0.08	-0.27	-0.24	-0.11
15	-0.09	-0.38	-0.35	-0.16
20	-0.12	-0.51	-0.48	-0.20
25	-0.13	-0.61	-0.61	-0.24
30	-0.16	-0.70	-0.74	-0.27
35	-0.21	-0.79	-0.89	-0.29
40	-0.25	-0.85	-0.99	-0.31
45	-0.26	-0.82	-1.02	-0.32
50	-0.32	-0.80	-1.04	-0.33
55	-0.34	-0.74	-1.00	-0.33
60	-0.35	-0.64	-0.93	-0.32
65	-0.35	-0.55	-0.85	-0.31
70	-0.36	-0.52	-0.78	-0.29
75	-0.32	-0.44	-0.68	-0.26
80	-0.33	-0.41	-0.61	-0.23
85	-0.26	-0.27	-0.47	-0.19
90	-0.24	-0.22	-0.38	-0.15
95	-0.19	-0.20	-0.31	-0.10
100	-0.12	-0.12	-0.20	-0.05

Figure 1: Effects addition puts to portfolios of stocks and hedge funds

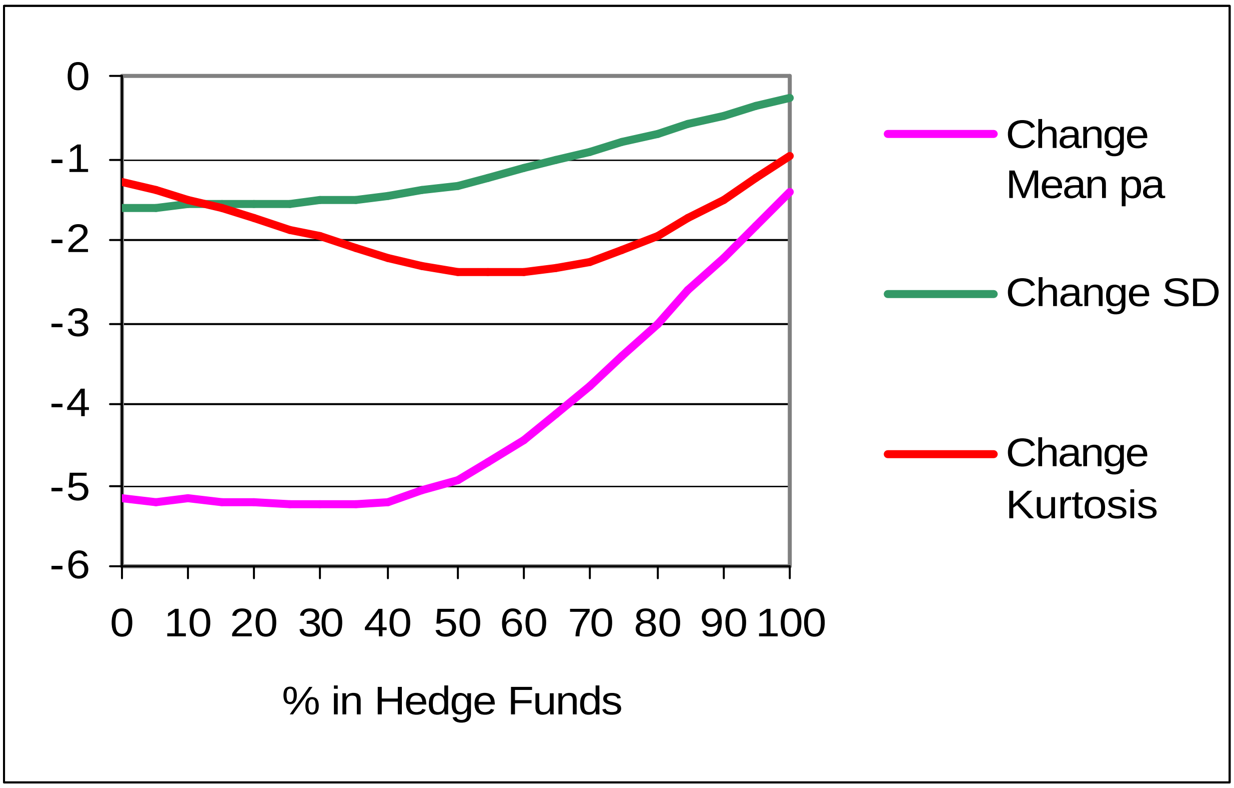

Figure 2: Effects addition puts and leverage to portfolios of stocks and hedge funds

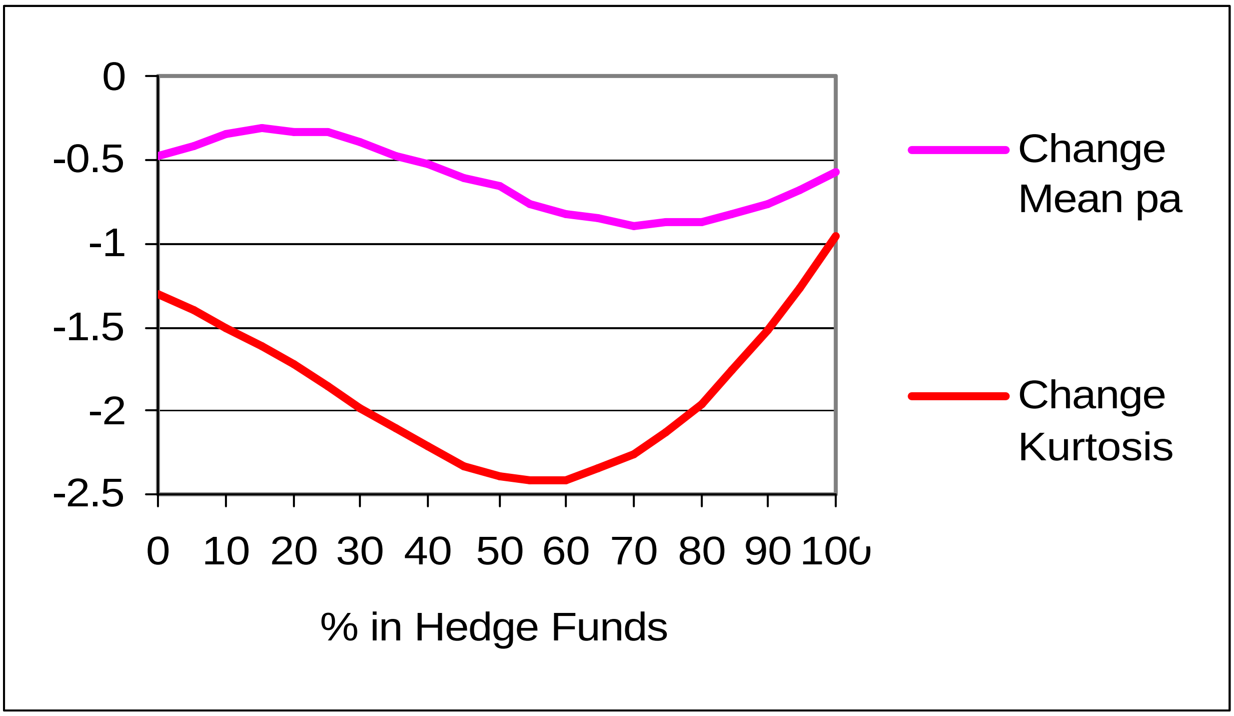

Figure 3: Effects addition puts to portfolios of stocks, bonds and hedge funds

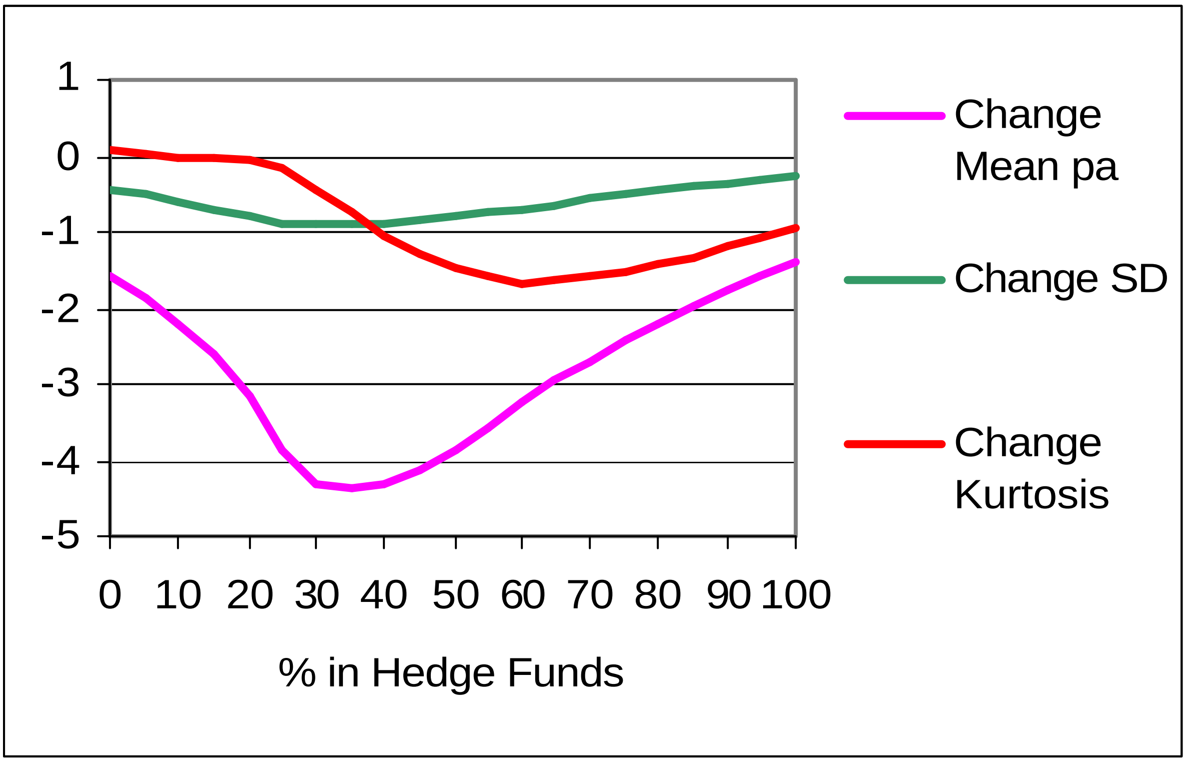

Figure 4: Effects addition puts and leverage to portfolios of stocks, bonds and hedge funds

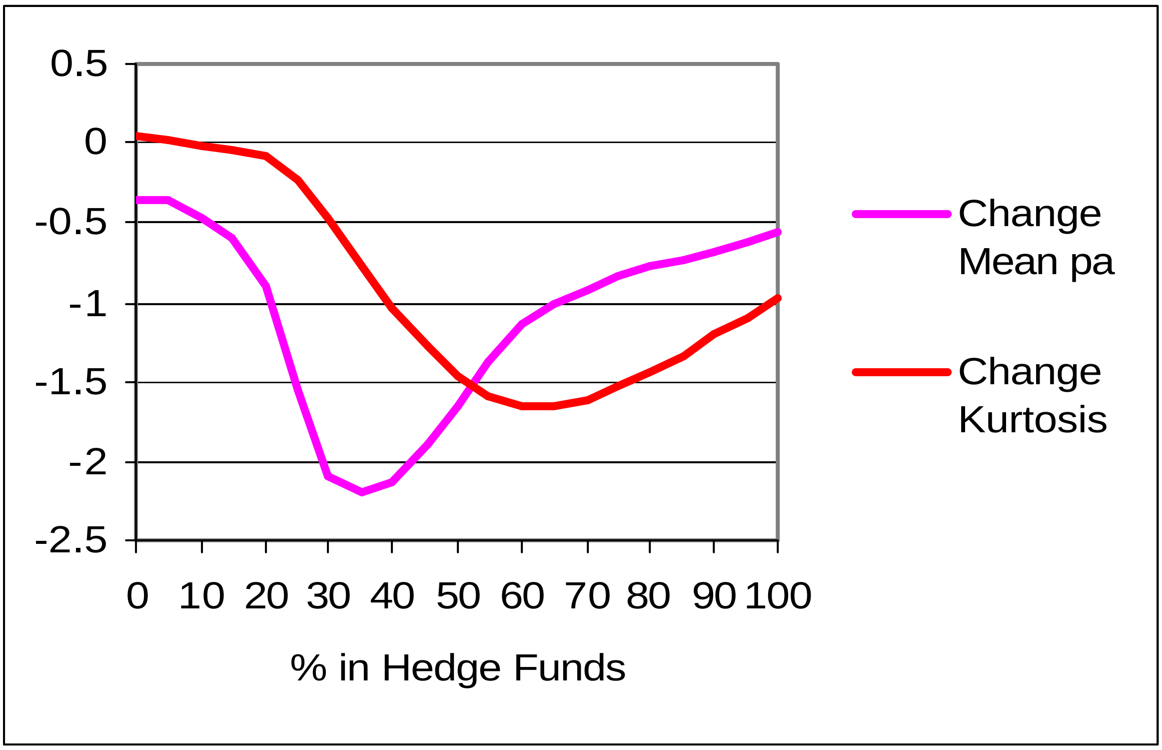

Figure 5: Frequency distribution monthly returns on portfolio of 40% stocks, 40% bonds and 20% hedge funds over period June 1994 – May 2001

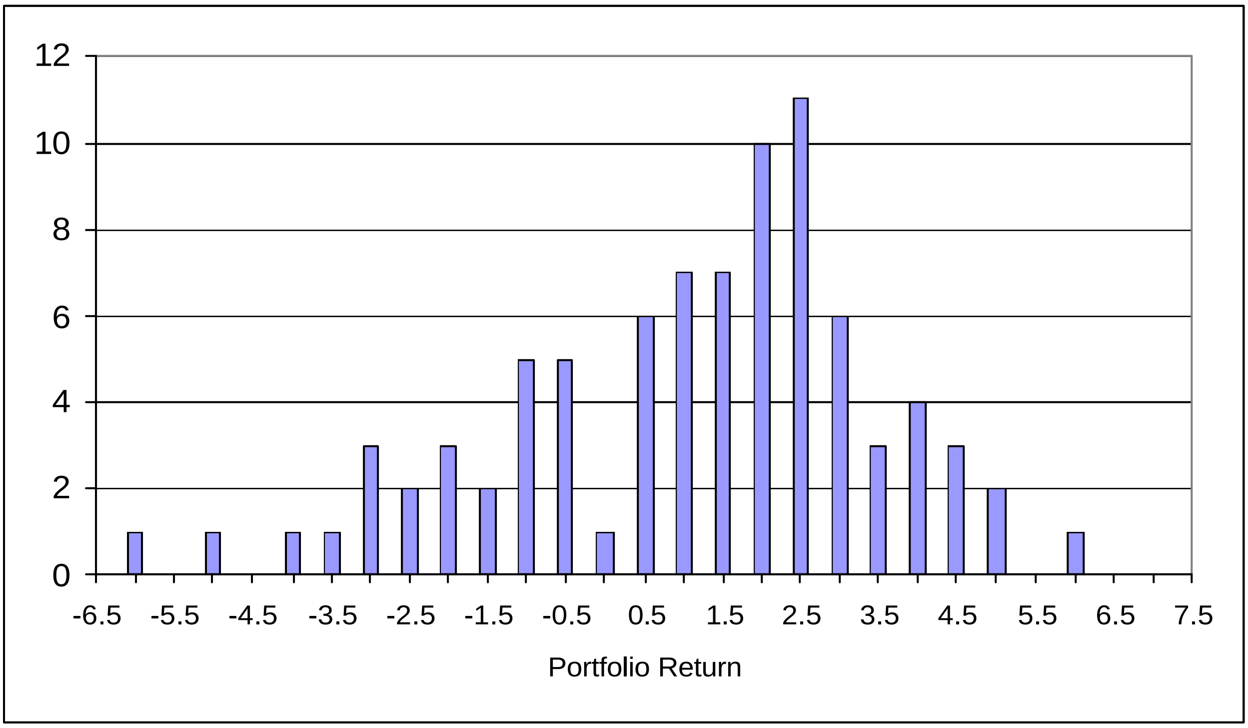

Figure 6: Frequency distribution monthly returns on portfolio of 60% stocks, 60% bonds, 30% hedge funds, 2% puts and -52% cash over period June 1994 – May 2001

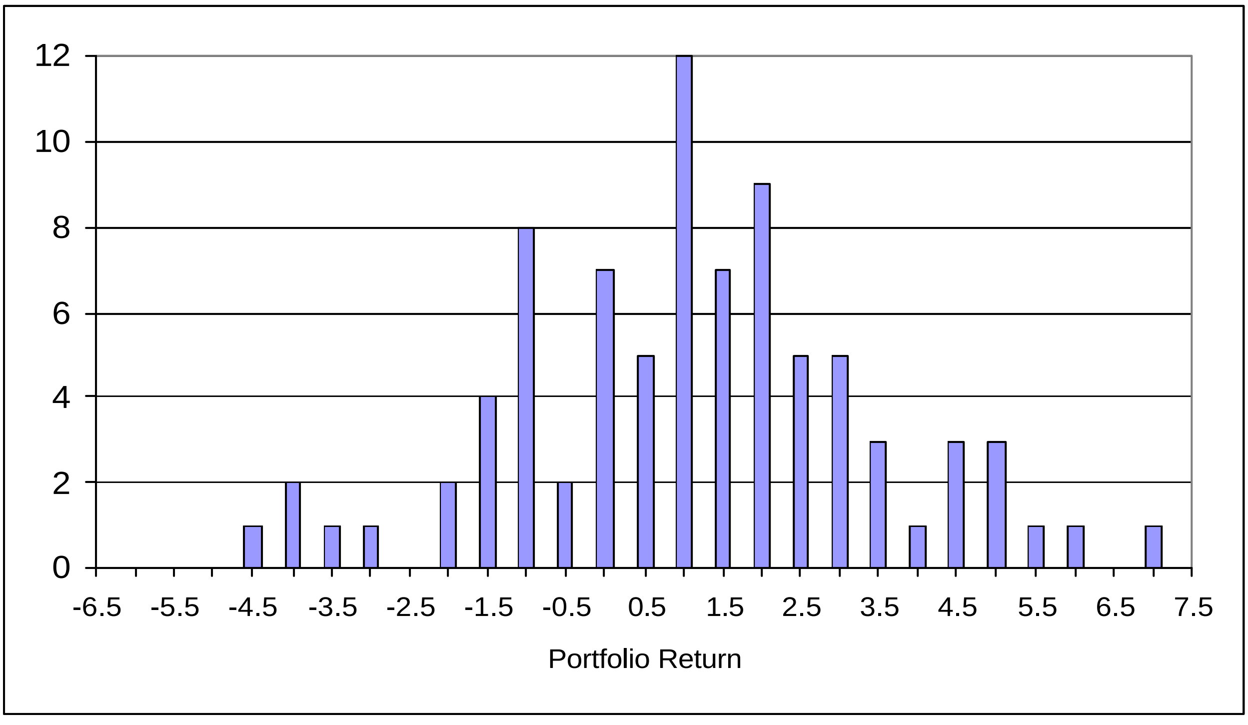

Figure 7: Effects addition puts and leverage to portfolios of stocks, bonds and hedge funds with –0.33 skewness target for different interest rates and expected stock returns